Fibonacci, Golden Section Resources

For a reference to the use of the Fibonacci series in music, see Jonathan Kramer, "The Fibonacci Series in Twentieth-Century Music," *Journal of Music Theory*, 17/1 (1973). Roy Howat's *Debussy in Proportion: A Musical Analysis* (Cambridge: Cambridge University Press, 1983) deals with Golden Section and Fibonacci issues. It has a wonderful Japanese painting on the cover, which is proportioned according to the Section.

The Three Hierarchies

The three hierarchies concern simultaneous evolving strata in a piece of music.

Type 1 is where strata are sorted according to their general speed. The slowest-going strata is at the highest level. There is no necessity of having attack points in one strata coincide with those on another.

In type 2, we have type 1 with a further constraint. Each time-point in a higher level strata must coincide with time-points on the lower, faster-going strata; this feature is often called *well-formed* in liguistics.

Type 3 is cyclic and has the features of a type 2 hierarchy plus the fact that the successive durations between attack points on each level have to express the same sequence of time ratios.

See examples of each type on the next page:

```
type 1:

level 1    *         * *       *                *     *       *
level 2   *       * *   *      *       * * *     *    *    * *
level 3  *  * *  * * * * *** * ***  * ** ** *** * ** ***

type 2:

level 1  *     * *     *           *     *         *
level 2  *   * * *     *  *     *  *  *  *        **
level 3  * * * * *     *  *  *  * ** *****    * *  * *

type 3a (regular):

level 1  *       *       *       *       *       *  (ratios 1:1:1: etc.)
level 2  *   *   *   *   *   *   *   *   *   *   *
level 3  * * * * * * * * * * * * * * * * * * * * *

type 3b (irregular, but periodic):

level 1     *     *   *       *   *       *   (ratios 1:2, 2:1) 8,4,8,4 etc.
level 2   * *   * *   * *   * *   * *   * *                    4,2,4,2 etc.
level 3  ** ** ** ** ** ** ** ** ** ** ** **                   2,1,2,1 etc.
```

More complex type 3b hierarchies are generated by taking time-points of the series 2^n, mod 2^{n-1}. If 2^{n-1} is not prime, the series can be taken mod $(2^{n-1})/k$, where k is a factor of 2^{n-1}, producing a more complex result. Another way to generate these series is via the orbits of a cyclic group generated by multiplying by n mod m. When m is 2^{n-1} this method is the same as the first.

```
level 1  *           *       *   *   *           *       *   *
level 2  *     *   * * *     *   * * *     *   * * *     *   *  *
level 3  *  * ***  * ***  * ***  * ***  * ***  * ***  * ***  * **
```

Each level has the same sequence of ratios of its pairs of successive durations, namely: 3:2 2:1 1:1 1:1.

```
level 3 is 3,2,1,1
level 2 is 6,4,2,2
level 1 is 12,8,4,4
```

This example was generated by the mult-by-2 cyclic group mod 7 from the orbits: (0) and (356). (I.e., from the above definition, m = 7 and n = 2)