Elementary Twelve-Tone Theory

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Definitions

row. A pc segment of all pcs without duplication. (In Princetonian twelve-tone literature “rows” are called “sets”; and our “pcsets” are called “collections.”)

row names. We use any capital letter to stand for a row. P is often used. (P need not start with pc 0.) Transformations of P are notated as operations to the left of P. For instance, given operations G and H on P, we write HGP. We first perform G on P, then H on that.

IP is the inversion of P.
T_nP is the transposition of P by n.
T_nIP is the transposed inversion of P with the index number n. (We do not use IT_nP as a row name (in any case, $IT_nP = T_{-n}IP$).
RP is the retrograde of P
RT_nIP is the retrograde of the transposed inversion of P.
Note: we write the R operation leftmost in a row name.

We do not use the row names P_n, I_n, R_n and RI_n.

P_a denotes the a^{th} pc in P (with P_0 being the first pc of P.)

pitch aspect A representation of a twelve-tone row in the standard way: the pcs of the row are written in the order of their associated order-numbers.

order aspect A representation of a twelve-tone row in which the order-numbers are written in order of "ascending" pitch-class.

ordered intervals in rows. The ordered (or directed) interval between P_x and P_y is given by $P_y – P_x$.

row-class A set of rows related by a canonical group of operators.

twelve-tone system A row class whose canonical group is the serial group (the 48 T_n, T_nI, and retrograde operations).

Types of rows

all-interval row A row whose INT has each directed (ordered) pc interval from 1 to B once and only once.

all-trichord row A ten-trichord row that excludes SC 3-10[036] and 3-12[048].

multiple order-number function row (MOF) A row R whose pcsegments are merged (to within retrograde) in row S, and R and S are in the same row-class.

supersaturated set-type rows A row in which every pc belongs to two imbricated row segments of the same set-class (or of two ZC-related set-classes).

self-deriving row A row that can generate one or more combination matrices such that each column of the matrix can be ordered as a row in the row-class of the generating row.

Mosaics

mosaic A partition of the aggregate.

mosaic-class The set of all mosaics related by T_n or T_nI.

Pc and Order Number Operations

TTO A canonical operator that takes the form T_nM_m where n = 0 to B and m = 1, 5, 7, or B. There are 48 TTOs.
$T_0 = T_0M_1$; $T_n = T_nM_1$
$I = M_B$; $T_nI = T_nM_B$,
$M = M_5$; $T_nM = T_nM_5$,
$MI = IM = M_7 = M_BM_5$; $T_nMI = T_nM_7$.

order-number TTOs Order-number transpositions (rotations), combined with the order-number multiplication operator m_X, where x = 1, 5, 7, or B. (Note that the R operation = T_BI on order numbers, and that rotation by n positions = T_n on order numbers.)

Types of Row Classes

segment group system (SeG system) A partition of the total number of pcsegments of a particular kind K such that if Y = GX, then Y, X $\in$ SeG(X), for all segments X and Y and the operation G $\in$ canonical group **G**.

The *(classical) twelve-tone system* is the SeG system with the serial group as its canonical group. K is a twelve-tone row.

general row system The SeG system of twelve-tone rows whose canonical group is the set of TTOs together with the order-number TTOs. K is a twelve-tone row.

grand row system The SeG system of twelve-tone rows whose canonical group is the pc T_n and T_nI operations together with the set of rotations and retrogrades (order-number T_n and T_nI). K is a twelve-tone row.

Functions on rows: INT

INT(P) A list of the adjacent ordered intervals of P. It consists of 11 places; the first place is given by P_1-P_0, the second by P_2-P_1, etc. up to P_B-P_A.

Ex: The INT(P) = <134BA463413> where P = <014875936AB2>.

Note the following identities, which follow from the fact that the retrograde of an ordered interval is the same interval as its inversion.

For any n:
INT(T_nP) = INT(P)
INT(T_nIP) = I(INT(P))
INT(RT_nP) = RI(INT(P))
INT(RT_nIP) = R(INT(P))

INT_n(P) A list of the ordered intervals of P n order numbers apart. It consists of 12-n places; the first place is given by P_n-P_0, the second by P_{n+1}-P_1, etc. up to P_B -P_{11-n}. The INT(P) = INT_1(P).

Exx: INT_2(P) = <47392A9754> where P = <014875936AB2>.
INT_5(P) = <58BA365> where P = <014875936AB2>.
INT_A(P) = <B1> where P = <014875936AB2>.

INT_B(P) = <z> where z is the directed interval between the first and last pc of P.
INT_0(P) = <000000000000> for all rows.

Functions on rows: BIP

$BIP_n(P)$ A list of the set-classes of imbricated segments of size n in P. It consists of 13-n places; the first place is given by the set-class of the pcs from P_0 to P_n, the second place is given by the set-class from P_1, to P_{n+1}, etc. up to the 12-n^{th} place by the set-class of the pcs from P_{12-n} to P_B. We use the second part of Forte's two numeral names to denote the set-classes. (Note, a slightly different definition is given for BIP_n in my *Class Notes for Atonal Theory*.)

Note $BIP_2(P)$ is an 11-place list of the adjacent ics (unordered interval-classes) in P. Note: $BIP_2(P)$ is not the same as $INT_1(P)$.

Identities:

$BIP_n(P) = BIP(T_xP) = BIP(T_xIP)$
$R(BIP_n(P)) = BIP(RT_xP) = BIP(RT_xIP)$

Example: Let P = <014875936AB2>

$BIP_2(P)$ = <13412463413> (P's first ic is 1; the second ic = 3, etc.)

$BIP_3(P)$ = <3 11 3 2 6 8 10 11 4 3> (The set-class of the first ordered trichord of P <014> is 3-3[014]; the set-class of the second ordered trichord of P <148> is a member of 3-11[037]; the last (10^{th}) ordered trichord of P <AB2> is a member of 3-3[014]; etc.)

$BIP_6(P)$ = < 19 15 4 2 10 43 44> (The set-class of the first ordered hexachord of P <014875> is a member of 6-19[013478]; the set-class of the middle ordered hexachord of P <875936> (including pcs from P_3 to P_8) is a member of 6-2[012346]; the last ordered hexachord of P <936AB2> is a member of 6-44[012569]; etc.

We can succinctly list the BIP$_n$(<014875936AB2>) from n = 3 to 11 as follows:

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 [3]  3 11  3  2  6  8 10 11  4  3
 [4] 19 18  3  2 21 12 18 20 19
 [5] 22 16  3  9  8 18 38 21
 [6] 19 15  4  2 10 43 44
 [7] 21 13  1  2 13 22
 [8] 19  2  1  2 19
 [9]  3  2  1  3
[10]  3  2  1
[11]  1  1
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